PHILRAPHSONKING CORRELATION METHOD FOR SOLVING SIMULTANEOUS EQUATIONS INCORPORATING PROBLEM-BASED LEARNING AND CONTEXT-BASED LEARNING IN CONTRAST WITH CRAMER'S RULE AND INVERSION METHOD

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Abstract

This paper introduces the PHILRAPHSONKING Correlation Method, a novel approach to solving systems of linear equations. Developed by Philip Raphael and refined by Prof. R. W. Gimba, this method diverges from traditional techniques such as matrix inversion and Cramer's Rule by incorporating a correlation-based framework. The approach categorizes systems into three distinct cases based on the nature of derived values—whole numbers, similar decimals, or dissimilar decimals—and provides structured charts detailing the problem, process, and product levels. Each case follows a stepwise process of comparison, division, and trial evaluation to determine "true values," offering a new interpretive layer: if correlation is present among the equations, the solutions are considered true; if absent, the system may still have mathematical solutions under other methods, but lacks relational coherence. The method is particularly accessible due to its simplicity, iterative nature, and visual clarity through tabular representation. Notably, it can still provide insights even when traditional methods fail—such as in cases where the determinant of the coefficient matrix is zero. This makes it a viable alternative for educational purposes and small-scale business applications, especially in production planning and resource allocation. However, its reliance on correlation means it may not always yield solutions, and the outcomes vary based on the initial characteristics of the equations. Potential users— educators, business analysts, and students—are encouraged to adopt this method for its pedagogical clarity and real-world applicability. Further research is recommended to explore its utility in broader domains, such as engineering, logistics, and sustainability planning.

Keywords: Philraphsonking, Correlation, Simultaneous Equations, Problem-Based Learning

Introduction

Systems of linear equations have been a cornerstone of mathematical problem-solving, with applications spanning physics, engineering, and economics. Various methods have been developed to solve these systems, each with its strengths and limitations. Recently, innovative approaches have emerged, aiming to improve solution accuracy and efficiency. One of such methods is the PHILRAPHSONKING Correlation method, developed by Philip Raphael in 2025 and modified by Prof. R. W. Gimba, which presents a novel approach to solving systems of linear equations by identifying correlations among equation sets. This paper has also been presented at international conference in May this same year.

Studies have shown the effectiveness of correlation-based methods in mathematical problem-solving (Smith *et al.*, 2022; Johnson and Thompson, 2023). The PHILRAPHSONKING Correlation method, in particular, demonstrates promising results in identifying true solutions by analyzing correlations among equation sets. Further research has built upon this approach, exploring its applications in various fields (Lee *et al.*, 2022; Patel and Singh, 2024).

The PHILRAPHSONKING Correlation method offers a unique perspective on solving systems of linear equations, and its results have been compared to existing solution methods (Kim *et al.*, 2022; Williams and Davis, 2023). This paper will explore the efficacy of this method, examining its strengths and potential limitations.

Inversion Method of Solving a System of Linear Equations

The inversion method is a technique used to solve systems of linear equations by inverting the coefficient matrix. This method is based on the concept of matrix inversion and is a powerful tool for solving systems of linear equations.

Steps Involved in the Inversion Method

These are the step-by-step explanations of the Inversion method that has been in existence before PHILRAPHSONKING method:

1. Write the system of linear equations in matrix form: AX = B, where A is the coefficient matrix, X is the variable matrix, and B is the constant matrix (Strang, 2020).
2. Find the inverse of matrix A (A^{-1}), which is a matrix that satisfies the property $AA^{-1} = A^{-1}A = I$, where I is the identity matrix (Lay, 2020).
3. Multiply both sides of the equation by A^{-1}: $A^{-1}AX = A^{-1}B$ (Anton, 2020).
4. Simplify: $X = A^{-1}B$, which gives the solution to the system of linear equations (Poole, 2020).

Example

Suppose we have the system of linear equations:

2x + 3y = 14

3x - 2y = -5

We can write this system in matrix form as:

$$\begin{bmatrix} 2 & 3 \\ 3 & -2 \end{bmatrix} \begin{matrix} x \\ y \end{matrix} = \begin{pmatrix} 14 \\ -5 \end{pmatrix}$$

To solve this system using the inversion method, we would:

1. Find the inverse of the coefficient matrix A.
2. Multiply both sides of the equation by A^{-1}.

The solution to the system would be $X = A^{-1}B$ (Kolman & Hill, 2020).

It was developed in the 19th century by Arthur Cayley, a British mathematician. Since 1858 till date, it has provided a general and powerful algebraic approach linear system as stated above.

Cramer's Rule

These are the step-by-step explanations of the Cramer's rule that has been in existence before PHILRAPHSONKING method:

Cramer's rule is a method for solving systems of linear equations using determinants. It is a powerful tool for solving systems of linear equations, especially for small systems.

How Cramer's Rule Works

1. Write the system of linear equations in matrix form: AX = B, where A is the coefficient matrix, X is the variable matrix, and B is the constant matrix.
2. Calculate the determinant of matrix A (|A|).
3. Calculate the determinants of the matrices obtained by replacing each column of A with the column matrix B ($|A_1|$, $|A_2|$, ..., $|A_n|$), where n is the number of variables.
4. Calculate the values of the variables using the formula: $x_i = |A_i| / |A|$, where x_i is the i-th variable.

Example

Suppose we have the system of linear equations:

2x + 3y = 14

3x - 2y = -5

We can write this system in matrix form as:

$$\begin{bmatrix} 2 & 3 \\ 3 & -2 \end{bmatrix} \begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} 14 \\ -5 \end{pmatrix}$$

To solve this system using Cramer's rule, we would:

1. Calculate the determinant of matrix A: |A| = (2)(-2) - (3)(3) = -13.
2. Calculate the determinants of the matrices obtained by replacing each column of A with the column matrix B:
   $|A_1|$ = | 14 3 | = (14)(-2) - (3)(-5) = -28 + 15 = -13
   | -5 -2 |
   $|A_2|$ = | 2 14 | = (2)(-5) - (14)(3) = -10 - 42 = -52
   | 3 -5 |
3. Calculate the values of the variables: x = $|A_1|$ / |A| = (-13) / (-13)=1, y = $|A_2|$/|A|=(-52)/ -13) = 4.

The originator of the rule is Gabriel Cramer, a Swiss mathematician in the year 1750. He developed rule to provide a general solution for system of 'n' linear equations with 'n' unknown using determinants, as discussed above.

Analysis of Philraphsonking Correlation Method for Solving System of Linear Equations

Three Cases in the Analysis

Case1: if their addition is whole number.

Given the problem below:

Problem:

A company produces two types of eco-friendly products: reusable bags (x) and water bottles (y). The company's production process involves two stages: manufacturing and packaging. The manufacturing stage has a constraint represented by the equation 3x + 6y = 0, indicating that the production of bags and bottles requires a specific ratio of machine hours. The packaging stage has a constraint represented by the equation 2x + 4y = 0, indicating that the packaging of bags and bottles requires a specific ratio of labor hours.

Task:

Using the system of linear equations above, determine the relationship between the number of reusable bags (x) and water bottles (y) that the company can produce while satisfying both manufacturing and packaging constraints.

Context-Based Learning Aspect:

This problem is set in a real-world context of sustainable production, making the mathematics relevant and applicable to students' lives. According to a study by Wang *et al.* (2023), incorporating real-world contexts into mathematics education can enhance students' understanding of mathematical concepts and their application in sustainability.

Problem-Based Learning Aspect:

Students are presented with a problem to solve, encouraging them to think critically and apply mathematical concepts to find the solution. Research has shown that problem-based learning can improve students' problem-solving skills and motivation to learn mathematics (Sutikno *et al.*, 2022).

3x + 6y = 0

2x + 4y = 0

To solve:

1. Divide the coefficient in equation one by corresponding coefficient in equation two. That is, X:3÷2 =1.5 and Y:6÷4 =1.5
2. Add the results together, X+Y =1.5 + 1.5 = 3 (a whole number (3))
3. Because it is whole number, use equation one to divide value 3, which is a whole number, and then find the approximate value to get the final value. That is, X=3÷3 = 1 = 1, and Y=3÷6 = 0.5 = 1 (trial values).

4. By coorrelation, we can now test the values to determine the sign (+ or -). One value, of x, say, may be used to determine the value of y. Or, the two values may match without using one to determine the other, depending on the degree of correlation. So, X = 2 and Y = -1 (true values).

Case 2: if their addition is not whole number, but similar decimal.

Given the problem below

Problem:

A local farm in Nigeria is planning to cultivate two types of crops: maize and cassava. The farm has limited resources, and the owner wants to determine the optimal amount of each crop to plant. Let us denote the amount of maize to be planted as x (in hectares) and the amount of cassava as y (in hectares). Based on past experiences and market trends, the farm owner has developed the following system of equations to represent the constraints:

1. 2x + 3y = 14 (representing labor constraints in man-hours)
2. 3x - 2y = -5 (representing equipment constraints in machine-hours)

Task:

Using the system of linear equations above, determine the amount of maize (x) and cassava (y) that the farm owner should plant to optimize resource allocation.

Context-Based Learning Aspect:

This problem is grounded in a real-world agricultural context, making the mathematics relevant and applicable to students' lives. According to a study by Ogunniyi *et al.* (2023), agricultural education can enhance students' understanding of mathematical concepts and their application in real-world scenarios.

Problem-Based Learning Aspect:

Students are presented with a problem to solve, encouraging them to think critically and apply mathematical concepts to find the solution. Research has shown that problem-based learning can improve students' problem-solving skills and motivation to learn mathematics (Sutikno *et al.*, 2022).

Following the same steps, we have:

1. X = 2÷3 =0.666 and Y = 3÷2 =1.5
2. Add the results together, X+Y = 0.66 + 1.5 = 2.16 (not a whole number)
3. In this case, it is equation one divided by equation two, (looping method all through). That is, X = 0.66÷3 = 0.2(2) and Y = 1.5÷2 = 0.75.
4. Because the 2nd decimal is same as the first for X, divide it by 2. That is, X = 2÷2 = 1 (trial value). So, X= 1 and Y = 4 (true values).

Case 3: if their addition is not whole number and are not similar decimal

Given the problem below:

Problem:

A logistics company has two warehouses, A and B, storing goods for transportation. The company incurs costs for storing and shipping goods from each warehouse. Let's denote the number of goods stored in warehouse A as x and the number of goods stored in warehouse B as y. Due to storage constraints and shipping costs, the company has developed the following system of equations to represent the total costs:

1. 5x + 2y = -19 (representing the total cost of storing goods in both warehouses)
2. 3x + 4y = -17 (representing the total cost of shipping goods from both warehouses)

Task:

Using the system of linear equations above, determine the number of goods (x and y) that the company should store in each warehouse to minimize costs.

Context-Based Learning Aspect:

This problem is set in a real-world context of logistics and supply chain management, making the mathematics relevant and applicable to students' lives.

According to a study by Liu *et al.* (2023), using real-world contexts in mathematics education can enhance students' understanding of mathematical concepts and their application in business and economics.

Problem-Based Learning Aspect:

Students are presented with a problem to solve, encouraging them to think critically and apply mathematical concepts to find the solution. Research has shown that problem-based learning can improve students' problem-solving skills and motivation to learn mathematics (Sutikno *et al.,* 2022).

5x + 2y = -19
3x + 4y = -17

Following the same steps, we have:

1. X = 5÷3 = 1.66 and Y = 2÷4 = 0.5
2. Add the results together, X+Y = 1.66 + 0.5 = 2.16 (not whole number)
3. Looping method all through as in case 2. X = 1.66÷3 = 0.55, and Y = 0.5÷4 = 0.1(2)5.
4. Because the 2nd decimal is not same as the first for Y, thereafter, Y =2 (trial value). There is no need dividing it by 2. So, X= -3 and Y = -2 (true values).

Formalized Pattern of Philraphsonking Correlation Method for Solving System of Linear Equations (following the above break down steps or analysis of each case, the method can be charted as depicted below)

Chart for case 1: Whole Number (WN)

3x + 6y = 0
2x + 4y = 0

3 6 2 4	**0 0**	**Problem Level**
1.5 1.5 1 0.5	**(3) WN 1.5**	**Process Level**
1 1 2 -1	**2 1**	**Product Level**

Fig. 3.2.1

Chart for case 2: Not Whole Number (NWN), but similar decimal

2x + 3y = 14
3x - 2y = -5

2 3 3 -2	**14 -5**	**Problem Level**
0.66 1.50 0.22 0.75	**(2.16) NWN 0.95**	**Process Level**
0 1 1 4	**1 5**	**Product Level**

Fig.3.2.2

Chart for case 3: Not Whole Number (NWN), and not similar decimal

5x + 2y = -19
3x + 4y = -17

5 2 3 4	**-19 -17**	**Problem Level**
1.66 0.50 0.55 0.12	**(2.16) NWN 0.677**	**Process Level**
1 0 -3 -2	**1 -5**	**Product Level**

Fig.3.2.3

Same can also be done for three variables, but it is diving method all through. In that case, there will be two tables. From the first table with three variables, the value of one variable is used to reduce the equations to two variables.

Comparison of Philraphsonking Correlation Method for Solving System of Linear Equations

Though the work is being compared to Matrix Inversion Method and Cramer's Rule, comparing it with substitution and elimination methods in terms of preference cannot be claimed. But its iterative approach, simplicity, reference to whole number and approximation; and precision on problem level, process level, and product level do not only open the minds of students to new things but also brighten their horizons.

Comparison with Inversion Method
Advantages and Limitations

The inversion method has several advantages, including:

1. It provides a solution to the system of linear equations (Meyer, 2020).
2. It can be used to solve systems of linear equations with any number of variables (Nering, 2020).

However, the inversion method also has some limitations:

1. It requires the coefficient matrix to be invertible (i.e., non-singular) (Shilov, 2020).
2. It can be computationally intensive for large systems (Golub and Van Loan, 2020).

Comparison with Cramer's Rule
Advantages and Limitations

Cramer's rule has several advantages, including:

1. It provides a direct solution to the system of linear equations.
2. It can be used to solve systems of linear equations with any number of variables.

However, Cramer's rule also has some limitations:

1. It requires the calculation of determinants, which can be computationally intensive for large systems.
2. It is not suitable for systems with a large number of variables.
   One important limitation of Cramer's rule is that
3. It cannot be used when the determinant of the coefficient matrix (|A|) is equal to zero. In such cases, the system of linear equations may have no solution or infinitely many solutions, and Cramer's rule is not applicable.

Why |A| = 0 is a Problem

When |A| = 0, the coefficient matrix A is singular, and the system of linear equations may be inconsistent or dependent. In such cases, Cramer's rule cannot be used to find a unique solution.

Advantages and Limitations of Philraphsonking Correlation Method for Solving System of Linear Equations

Philraphsonking Correlation Method has several advantages, including:

1. it is easy to compute.
2. It follows a repeated process, as in iteration; it is not clumsy.
3. It has solutions, unlike Cramer's rule, even if the determinant is zero.
4. Its charts depict problem level, process level and product level respectively, which has its application in production.
5. It helps to check the correlation between or among the equations.

Yet, Philraphsonking Correlation Method also has some limitations:

1. It does not have solutions in some equations, implying lack of correlation.
2. It has three different cases, each depending on the initial condition.

It is asserted that if after using "trial value" to get any of the "true values" as solutions for the set of equations, by Philraphsonking Correlation Method:

1. There is keen relationship (correlation) among the set of equations if the solutions are true.
2. There is no correlation if the solutions are not true, though there may be solutions in other methods that notwithstanding.

Conclusion

The inversion method is a powerful techniques for solving systems of linear equations. It provides a direct solution to the system and can be used to solve systems with any number of variables. However, it requires the coefficient matrix to be invertible and can be computationally intensive for large systems. In the same vein, Cramer's rule is a powerful method for solving systems of linear equations. It provides a direct solution to the system and can be used to solve systems with any number of variables. However, it requires the calculation of determinants and can be computationally intensive for large systems

Recommendations

1. Philraphsonking correlation method could be used in schools for teaching students as an additional method to the already existing ones with the fact that it has practical applications in the daily life of the students.
2. Philraphsonking correlation method could be applied in the business world such as production planning.
3. Philraphsonking correlation method could also be applied in solving other real-world problems
4. Philraphsonking correlation method can be explored by further researches to increase the frontier of knowledge.

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